A NONPERTURBATIVE CALCULATION OF BASIC CHIRAL QCD PARAMETERS WITHIN ZERO MODES ENHANCEMENT MODEL OF THE QCD VACUUM. II

V. Gogohia, Gy. Kluge and M. Prisznyák

RMKI, Department of Theoretical Physics, Central Research Institute for Physics,

H-1525, Budapest 114, P. O. B. 49, Hungary

Abstract

Basic chiral QCD parameters (the pion decay constant, the quark and gluon condensates, the dynamically generated quark mass, etc) as well as the vacuum energy density (up to the sign, by definition, the bag constant) have been calculated from first principles within a recently proposed zero modes enhancement (ZME) model of the true QCD vacuum. Our unique input data was chosen to be the pion decay constant in the chiral limit as given by the chiral perturbation theory at the hadronic level (CHPTh). In order to analyze our numerical results we set a scale by two different ways. In both cases we obtain almost the same numerical results for all chiral QCD parameters. Phenomenological estimates of these quantites as well as vacuum energy density are in good agreement with our numerical results. Complementing them by the numerical value of the instanton contribution to the vacuum energy density, we predict new, more realistic values for the vacuum energy density, the bag constant and the gluon condensate.

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I. INTRODUCTION

Let us begin the second part of our paper with the discussion of one of the most interesting feature of (dynamical chiral symmetry breaking) DCSB. As was underlined in the first part (hereafter referred to as I), there are only five independent quantities by means of which all other chiral QCD parameters should be calculated. For the sake of convenience, let us write down them together. They are:

$$F_{CA}^2 = \frac{3}{8\pi^2}k_0^2 z_0^{-1} \int_0^{z_0} dz \, \frac{zB^2(z_0, z)}{\{zg^2(z) + B^2(z_0, z)\}}, \tag{1.1}$$

$$m_d = k_0\{z_0 B^2(z_0, 0)\}^{-1/2}, \tag{1.2}$$

$$\langle \overline{q}q \rangle_0 = -\frac{3}{4\pi^2}k_0^3 z_0^{-3/2} \int_0^{z_0} dz \, zB(z_0, z), \tag{1.3}$$

$$\epsilon_q = -\frac{3}{8\pi^2}k_0^4 z_0^{-2} \int_0^{z_0} dz \, z \, \{\ln z \left[zg^2(z) + B^2(z_0, z) \right] - 2zg(z) + 2\}, \tag{1.4}$$

$$\epsilon_g = -\frac{1}{\pi^2}k_0^4 z_0^{-2} \times \left[18\ln(1 + \frac{z_0}{6}) - \frac{1}{2}z_0^2 \ln(1 + \frac{6}{z_0}) - \frac{3}{2}z_0 \right]. \tag{1.5}$$

Recall that $g(z)$ and $B^2(z_0, z)$ are explicitly given by (1.14) and (1.15) of I. It is instructive also along with them to write down definition (3.13) of I for DCSB scale, namely

$$\Lambda_{CSBq} = 2m_d. \tag{1.6}$$

So these final expressions which should be used to calculate chiral QCD parameters within our approach depend only on two independent quantities, namely: mass scale parameter k_0 and the constant of integration of dynamical quark SD equation of motion z_0. However, it follows from (1.2) that information on the parameter z_0 should be extracted again from m_d and the initial mass scale parameter k_0 itself, which characterizes the region where confinement, DCSB and other nonperturbative effects begin to play a dominant role (see below). So the second indepent parameter z_0 is reduced to the pair of the mass scale

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parameters k_0 and m_d. Despite the fact that in our consideration the initial mass scale parameter μ (characterizing the scale of nonperturbative effects) has been introduced by "hand", such a transformation of pair of independent parameters k_0 and z_0 into the pair of k_0 and m_d is also a direct manifestation of the phenomenon of the "dimensional transmutation" [1]. This phenomenon occurs whenever a massless theory acquires masses dynamically and it is a general feature of spontaneous symmetry breaking in field theories.

Let us emphasize once more that it generally characterizes our approach, in order to calculate numerically all chiral QCD quantities (considered here and others), that one needs only two independent (free) parameters, both having significant and clear physical sense. The above mentioned dynamically generated quark mass m_d, playing a role of the integration constant of the corresponding equation of motion (the quark SD equation) instead of z_0 because of the above mentioned "dimensional transmutation" and mass scale parameter k_0, responsible for a scale at which important nonperturbative effects begin to play a dominant role. Our calculation scheme is self-consistent because we calculate $n = 5$ independent physical quantities by means of $m = 2$ free parameters having clear physical sense, so condition of self-consistensy $n > m$ is satisfied. The general behaviour of all of our parameters as given by the relations (1.1-1.5) are shown in Figs. 1-5.

Our approach makes it possible to calculate all chiral QCD parameters (the ones considered here plus others) at any requested combination of m_d and k_0, but in order to analyse numerical results it is necessury to set a scale at which it should be done. We set a scale by two, at first sight, different ways but leading (see below) to almost the same numerical results within our calculation scheme.

Evidently, to set a scale in each case makes it possible to determine only one of the two free parameters in our calculations. In order to determine the second one we use the chiral value of the pion decay constant obtained by the chiral perturbation theory at the hadronic level (CHPTh) in Ref. 2, namely $F_\pi^o = (88.3 \pm 1.1)\ MeV$. This value is chosen as an input data in our numerical investigation of chiral QCD. The pion decay constant is a good experimental number since it is a directly measurable quantity as opposed, for

example, to the quark condensate. For this reason we may reliably estimate the deviation of the chiral values of physical quantities, which can not be directly measured, from their "experimental", phenomenologically determined values, when the chiral value of the pion decay constant is approximated by the experimental value.

In the above mentioned CHPTh (or equvivalently the effective field theory) there is a low energy constant B, determined by $\langle \overline{q}q \rangle_0 = -F^2 B$ and measures the vacuum expectation value of the scalar densities in the chiral limit. It is just this constant that determines the meson mass expansion in the general case. Indeed, in leading order (in powers of quark masses and e^2) from CHPTh, one has [3, 4]

$$M_{\pi^+}^2 = (m_u^0 + m_d^0)B \tag{1.7}$$

$$M_{K^+}^2 = (m_u^0 + m_s^0)B \tag{1.8}$$

$$M_{K^0}^2 = (m_d^0 + m_s^0)B. \tag{1.9}$$

Calculating (independently) the constant B, one then will be able to correctly estimate current quark masses m_u^0, m_d^0 and m_s^0 by using the experimental values of meson masses [5] in (1.7-1.9).

II. ANALYSIS OF THE NUMERICAL DATA AT A SCALE OF DCSB AT THE QUARK LEVEL

Let us begin by recalling that there exists a natural scale within our approach to DCSB. Indeed, at the fundamental quark level the chiral symmetry is spontaneously broken at a scale Λ_{CSBq} defined by (1.6). Therefore it makes sense to analyse our numerical data at a scale where DCSB at the fundamental quark level occurs. To this end, it is necessary only to simply identify mass scale parameter k_0 with this scale Λ_{CSBq}, i.e. to put

$$k_0 \equiv \Lambda_{CSBq} = 2m_d. \tag{2.1}$$

In other words, we will analyse our numerical results at a scale responsible for DCSB at the fundamental quark level. Evidently, this uniquely determines the constant of integration of

the quark SD equation. Indeed, from (2.1) and on account of (1.2), then it immediately follows that this constant is equal to $z_0 = 1.34805$. From the pion decay constant in the chiral limit, chosen as input data, and on account of this value for z_0, from (1.1) it yields the numerical value for k_0. This means that all physical parameters considered in our paper are uniquely determined. As it was mentioned above, it will be instructive to explicitly display our numerical results when the chiral value of the pion decay constant is approximated by the experimental value advocated in Refs. 6 and 7, namely $F_\pi^o = 92.42 \; MeV$, as well as by the standard value $F_\pi^o = 93.3 \; MeV$. Results of our calculations are displayed in Table 1.

Let us make a few concluding remarks. To set a scale by the way described in this section has the advantage that it is based on the exact definition (1.6) for a scale of DCSB at which analysis of the numerical data must be done. In general, it is not obvious that this scale Λ_{CSBq} and scale Λ_c, at which quark confinement occurs, should be of the same order of magnitude. Moreover, the information about Λ_c is hidden within this scheme of calculation. In order to reveal the raison d'etre for Λ_c and its relation to Λ_{CSBq}, let us set a scale in the way described in the next section.

III. ANALYSIS OF THE NUMERICAL DATA AT THE CONFINEMENT SCALE

In our approach there exists only one scale, denoted as μ or k_0 (separating, in general, the nonperturbative phase from the perturbative one), which is responsible for all the non-perturbative effects in QCD at large distances. If there is a close relation between quark confinement and DCSB (and we believe that this is so) then the scale of DCSB at the fundamental quark level (1.6) and the confinement scale Λ_c should be, at least, of the same order of magnitude. In other words, in our approach Λ_c should be very close to Λ_{CSBq}. This is in agreement with Monte Carlo simulations on the lattice which show that the deconfinement phase transition and the chiral symmetry restoring phase transition occur approximately at the same critical temparature [8], hereby confirming the close intrinsic link between these nonperturbative phenomena.

Unfortunately, neither the exact value of m_d nor of k_0 is known. For this reason, let us first reasonably assume that the dynamically generated quark masses, in any case,

$$300 \leq m_d \leq 400 \ (MeV), \tag{3.1}$$

should hold but otherwise they remain arbitrary. We believe that this interval covers all possible realistic values used for and obtained in various numerical calculations. The second independent parameter k_0 should be varied in the region of 1 GeV - the characteristic scale of low energy QCD. Varying independently these pairs of parameters m_d and k_0 numerically, one can calculate all chiral QCD parameters with the above derived formulae.

From the value of the pion decay constant in the chiral limit, as well as from the range selected first for m_d (3.1) and on account of (1.1) and (1.2), it follows that the momentum k_0 always should satisfy the upper and lower boundary value conditions, namely $691.32 \leq k_0 \leq 742.68 \ (MeV)$. The vacuum energy density contributions of the nonperturbative gluons (1.5) changes its sign in the range selected for m_d (3.1) and in this interval for k_0. Therefore it becomes positive and this should not be so because of the normalization condition (we normalize perturbative vacuum to zero). It is easy to show that this is result of that that the lower bound chosen for the dynamical generated quark mass in (4.1) is too low. Indeed, the vacuum energy density (1.5) vanishes at the critical point $z_0^{cr} = 1.45076$ (see Fig. 6). Then from (1.2) calculated at this point, it follows that

$$k_0 \leq 2.26 m_d. \tag{3.2}$$

Using this ineqaulity in additional, the vacuum energy density (1.5) will always be negative as it should be and it will become zero only at critical values determined as $k_0^{cr} = 2.26 m_d$. From the chosen interval for m_d (3.1) and the obtained interval for k_0, however, it follows that the ratio between the corresponding lower bounds $k_0/m_d = 691.32/300 = 2.3044$ does not satisfy the above obtained inequality (3.2), while this ratio for the corresponding upper bounds $k_0/m_d = 742.68/400 = 1.8567$ satisfies it. This explicitly shows that the lower bound for m_d in (3.1) was incorrectly chosen. The exact lower bound for m_d can be found from the k_0^{cr} as $742.68 = 2.26 m_d$, and (3.1) becomes

$$328.62 \leq m_d \leq 400 \ (MeV). \tag{3.3}$$

In the range determined by (3.3) and in the above obtained interval for k_0, the vacuum energy density (1.5) will be always negative because any combination (ratio) of k_0 and m_d from these intervals will satisfy inequality (3.2). But this is not the whole story yet. A new lower bound for m_d leads to a new lower bound for k_0 as well. Indeed, combine now this new lower bound (3.3) with the chiral value of the pion decay constant one obtains a new lower bound for k_0 as well.

As noted above, k_0 is regarded as a momentum which separates the nonperturbative phase (region) from the perturbative one. In the region obtained for k_0 the nonperturbative effects, such as quark confinement and DCSB, begin to play a dominant role. It is a region determining a scale at which confinement occurs. From now on let us call this scale for k_0 a confinement scale (in the chiral limit) and denote it Λ_c. So the final numerical values for the confinement scale are as follows

$$707 \leq \Lambda_c \leq 742.68 \ (MeV). \tag{3.4}$$

In intervals determined by (3.3) and (3.4) the vacuum energy density ϵ_g will be always negative (see Fig. 7).

It is worth noting that any value for Λ_c from interval (3.4) is possible but not any combination of Λ_c from interval (3.4) and m_d from interval (3.3) will automatically satisfy the value of the pion decay constant. Therefore it is necessary to adjust values of m_d from (3.3) for chosen value of Λ_c from interval (3.4) and vice versa (see Fig. 8). This means that m_d is in close relationship with Λ_c. Moreover, completing the above mentioned procedure, one finds that Λ_c is nearly the double of the generated quark mass m_d, i. e. $\Lambda_c \approx 2m_d$. This confirms that Λ_c and Λ_{CSBq} defined by (1.6) are nearly the same indeed. In the previous calculation scheme the adjusting procedure was automatically fulfilled because of the exact relation (2.1). Thus there is an intimate relationship between Λ_{CSBq} and Λ_c on the one hand and the double generated quark mass m_d on the other hand.

The interval (3.4) for possible values of Λ_c along with the new range for m_d (3.3) will uniquely determine numerically the upper and lower bounds for all other chiral QCD parameters considered here. Like in the previous case, our numerical results are shown in Table 2 (calculation scheme B), where the shorthand $\langle 0|G^2|0\rangle$ stands for the gluon condensate $\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$. Our numerical bounds for the vacuum energy density ϵ need additional remarks. We note that the bounds for ϵ is not the sum of bounds for ϵ_q and ϵ_g. The upper and lower bounds for ϵ_q are achieved at the upper and lower bounds for m_d (Λ_c) while for ϵ_g they are achieved at the lower and upper bounds of m_d (Λ_c).

Let us now prove the relation $\Lambda_c \approx 2m_d$. We have already learnt that correct values of k_0 belong to the interval for Λ_c (3.4). Then identifying k_0 with Λ_c in (3.2), one finally obtains

$$\Delta = \pm(-1 + \frac{\Lambda_c}{\Lambda_{CSBq}}) \leq 0.13, \tag{3.5}$$

where the positive sign corresponds to $\Lambda_c > \Lambda_{CSBq}$ and the negative one is valid when $\Lambda_c < \Lambda_{CSBq}$. In the derivation of this relation we used definition (1.6).

Finally it is worth underlining once more that besides good numerical results obtained in this section, we have established the existence of realistic lower bound for the dynamically generated quark masses. In each calculated case their numerical values are shown in Table 2. Thus one concludes that the vacuum energy density due to the nonperturbative gluons is sensitive to the lower bound for m_d. Another important result is that we have clearly shown that the confinement scale Λ_c and DCSB scale Λ_{CSBq} are nearly the same indeed.

IV. CONCLUSIONS AND DISCUSSION

Let us briefly compare our numerical results obtained from first principles with phenomenologically estimated values of the physical parameters considered here. An estimate of the quark condensate in Refs. 9 and 10,

$$\langle \overline{q}q \rangle_0^{1/3} = -(225 \pm 25)\ MeV \tag{4.1}$$

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is in good agreement with our values. It is worth noting here that QCD sum rules give usually the numerical values of physical quantities, in particular the quark condensate, approximately within an accuracy of (10-20)% (see, for example Ref. 11).

Our values for the current quark masses are also in good agreement with recent estimates from hadron mass splittings [12]

$$m_u^0 = (5.1 \pm 0.9) \ MeV,$$

$$m_d^0 = (9.0 \pm 1.6) \ MeV,$$

$$m_s^0 = (161 \pm 28) \ MeV \tag{4.2}$$

and QCD sum rules [13]

$$m_u^0 = (5.6 \pm 1.1) \ MeV,$$

$$m_d^0 = (9.9 \pm 1.1) \ MeV,$$

$$m_s^0 = (199 \pm 33) \ MeV, \tag{4.3}$$

see also reviews [14].

Here it is worth noting that from our numerical results (Tables 1 and 2) it follows that the constituent quark mass m_q should differ little from m_d. Apparently, the difference between them is of the order of a few per cent only of the displayed values of m_d. So without making a big mistake even for light quarks, it is possible to simply use m_d instead of m_q. Doing so one comes to the conclusion that the CHPTh value of the pion decay constant and the constituent quark model (CQM) with the value for the constituent quark mass $m_q = 362 \ MeV$ advocated by Quigg [15] are nearly in one-to-one correspondence within our calculation scheme (see Table 1). Moreover, from our numerical results (Tables 1 and 2) one can conclude that the dominant contributions to the values of all chiral QCD parameters as well as the vacuum energy density come from large distances, while the contributions from the short and intermediate distances can only be treated as small perturbative corrections.

The phenomenological analysis of the QCD sum rules [10] for the numerical value of the gluon condensate implies

$$\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle \simeq 0.012 \; GeV^4, \tag{4.4}$$

and using then (2.5) of I, one obtains the vacuum energy density as

$$\epsilon \simeq -0.003375 \; GeV^4. \tag{4.5}$$

In the random instanton liquid model (RILM) [16] of the QCD vacuum, for a dilute ensemble, one has

$$\epsilon = -\frac{9}{4} \times 1.0 \; fm^{-4} \simeq -0.003411 \; GeV^4. \tag{4.6}$$

The estimate of the gluon condensate within the QCD sum rules approach can be changed within a factor of two [10]. We trust our numerical results for the vacuum energy density much more than those of the gluon condensate. The former was obtained on the basis of the completely nonperturbative ZME model of the vacuum of QCD while the latter was obtained on account of the perturbative solution for the CS-GML β-function [10]. Let us also emphasize the one important fact that our calculation of the vacuum energy density is a calculation from first principles while in the RILM [16] the parameters characterizing vacuum, the instanton size $\rho_0 = 1/3 \; fm$ and the "average separation" $R = 1.0 \; fm$ were chosen to precisely reproduce traditional (phenomenologically estimated from the QCD sum rules) values of quark and gluon condensates, respectively.

We reproduce values (4.4-4.6), which are due to the instanton-type fluctuations only, especially well when the pion decay constant in the chiral limit was approximated by its experimental value. Moreover, our numerical results clearly show that the contribution to the vacuum energy density of the confining quarks with dynamically generated masses ϵ_q is approximately equal to ϵ_g, that is of the nonperturbative gluons. It is well known that in the chiral limit (massless quarks) tunneling is totally suppressed, i.e. the contribution of the instanton-type fluctuations to the vacuum energy density vanishes. It will be restored again in the presence of DCSB [17-19]. Thus, in principle, in the chiral limit and in the presence of DCSB, the total vacuum energy density should be the sum (as minimum) of these three quantities, i.e.

$$\epsilon_t = \epsilon_I + \epsilon_g + N_f \epsilon_q, \tag{4.7}$$

where ϵ_I describes the contribution of the instanton component to the vacuum energy density. We introduce also the explicit dependence on the number of different quark flavors N_f since ϵ_q itself is the contribution of a single confining quark. Of course, this should be valid for the non-chiral case as well. The distinction will be in the concrete values of each component, apart from, maybe, ϵ_g.

Let us now run a risk and make a few quantitative predictions. Indeed, not going into the details of the instanton physics (well described in Ref. 18) and in agreement with the authors of Ref. 10, it is worth assuming, for simplicity's sake, that the light and heavy quarks match smoothly. This allows one to choose for the instanton component of the vacuum energy density ϵ_I the average value between (4.5) and (4.6), i.e. namely $\epsilon_I \simeq -0.0034~GeV^4$. Then our predictions for more realistic values of the total vacuum energy density ϵ_t (for N_f light confining quarks with dynamically generated masses) and the corresponding values of the gluon condensate are listed in Tables 3 and 4, 5 for both calculation schemes A and B, respectively.

It is worth reproducing explicitly some interesting particular values of the total vacuum energy density and the corresponding values of the gluon condensate. Thus for a pure gluodynamics ($N_f = 0$) one has

$$\epsilon_t \simeq -0.005~GeV^4,$$
$$\langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle \simeq 0.0177~GeV^4, \tag{4.8}$$

and

$$-0.00661 \quad \leq \epsilon_t \leq \quad -0.003837~(GeV^4),$$
$$0.0136 \leq \langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle \leq 0.0235~(GeV^4). \tag{4.9}$$

Here and below the numbers correspond to the approximation of the pion decay constant in the chiral limit by its standard value.

For the more realistic case $N_f = 2$ one obtains

$$\epsilon_t \simeq -0.008 \ GeV^4,$$

$$\langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle \simeq 0.0283 \ GeV^4, \tag{4.10}$$

and

$$-0.00933 \quad \leq \epsilon_t \leq \quad -0.00724 \ (GeV^4),$$

$$0.0256 \leq \langle 0 | \tfrac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle \leq 0.0331 \ (GeV^4). \tag{4.11}$$

There exist already phenomenological estimates of the gluon condensate [20] as well as lattice calculations of the vacuum energy density [21] pointing out that the above mentioned standard values (4.4) and (4.5-4.6) are too small. Our numerical predictions are in agreement with these estimates though we think that their numbers for gluon condensate [20] are too big. At the same time, it becomes quite clear why the standard values are so relatively small, because they are due to the instanton component of the vacuum only.

In the above mentioned RILM [16], light quarks can propagate over large distances in the QCD vacuum by simply jumping from one instanton to the next. Within our model (see I and Ref. 20) propagation of all quarks is determined by the corresponding SD equations (due to the ZME effect) so that they always remain off mass-shell. Thus we need no picture of jumping quarks. Contrast to the RILM, we think that the main role of the instanton-like fluctuations is precisely to prevent quarks and gluons from the freely propagation in the vacuum of QCD. Running against instanton-like fluctuations quarks undergo difficulties in their propagation in the QCD vacuum which, in principle, is a very complicated inhomogenious medium. At some critical value of the instantons density the free propagation of the virtual quarks, apparently, become impossible, so they already never annihilate again with each other. Obviously, this is equivalent to the creation of the quark-antiquark pairs from the vacuum. From this moment nontrivial rearrangement of the vacuum can start. The above mentioned critical value can be reached when $\epsilon_I \simeq \epsilon_g + \epsilon_q$, i.e. when at least one sort of quark flavors is presented in the QCD true vacuum. On one hand, this is supported

by our numerical results for $\epsilon = \epsilon_g + \epsilon_q$. On the other hand, the numerical values for ϵ_I, as given by (4.5) or (4.6), also confirms this. In the realistic (nonchiral) case the instanton part, along with other contributions, may substantilly differ from those shown in Tables 1, 2, 3, 4 and 5.

The bag constant is defined as the difference between the energy density of the perturbative and the nonperturbative QCD vacuums. We normalize the perturbative vacuum to zero (I). So in our notations the bag constant becomes

$$B = -\epsilon_t \qquad (4.12)$$

(Not to be mixed with the CHPTh constant (1.7-1.9)). Our predictions for this quantity are also shown in Tables 3 and 4, 5 for each calculation scheme A and B, respectively. In fact, our values for the bag constant overestimate the initial MIT bag [23] volume energy by one order of magnitude. Nevertheless, we think that the introduction of this constant into physics was a main achievement of the bag model.

As in previous case, let us explicitly reproduce some interesting concrete values of the bag constant. For a pure gluodynamics ($N_f = 0$) it is:

$$B \simeq 0.005 \; GeV^4 \simeq (266 \; MeV))^4 \simeq 0.651 \; GeV/fm^3. \qquad (4.13)$$

For the more realistic case $N_f = 2$ the bag constant becomes

$$B \simeq 0.008 \; GeV^4 \simeq (300 \; MeV)^4 \simeq 1 \; GeV/fm^3. \qquad (4.14)$$

For simplicity's sake we reproduced its value obtained within the calculation scheme A only. It has been noticed in [24] that noybody knows yet how big the bag constant might be, but generally it is thought it is about 1 GeV/fm^3. The predicted value for $N_f = 2$ is in fair agreement with expectation.

REFERENCES

[1] S.Colemen and E.Weinberg, Phys.Rev., **D7** (1973) 1888;

D.J.Gross and A.Neveu, Phys.Rev., **D10** (1974) 3235

[2] P.Gerber and H.Leutwyler, Nucl.Phys., **B321** (1989) 387

[3] H.Leutwyler, Preprint BUTP-92/42. Rapporteur talk given at the 26 Int.Conf.
on High Energy Phys., Dallas, 1992;

H.Leutwyler, Preprint BUTP-91/26. Lectures given at the advanced Theor.Study
in Elem.Part.Phys., Boulder, Colorado, 1991

[4] G.Ecker, Preprint CERN-TH.6660/92. Lectures given at the
Cargese Summer School on Quantitative Part.Phys., Cargese;

A.Pich, Preprint hep-ph /9502366. To be published in Rep. Prog.Phys.

[5] Particle Data Group, Phys.Rev., **D45** (1992) S1

[6] B.R.Holstein, Phys.Lett., **B244** (1990) 83

[7] W.J.Marciano and A.Sirlin, Phys.Rev.Lett., **71** (1993) 3629

[8] Y.Iwasaki, K.Kanaya, S.Sakai and T.Yoshie, Phys.Rev.Lett., **69** (1992) 21

[9] L.J.Reinders, H.Rubinstein and S.Yazaki, Phys.Rep., **127** (1985) 1

[10] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys., **B147** (1979) 385, 448

[11] A.V.Smilga, Preprint BUTP-91/41, 1991

[12] S.Narison, Riv.Nuovo Cim., **10** (1987) N.2

[13] C.A.Dominguez and E.de Rafael, Ann.Phys., **174** (1987) 372

[14] T.Hatsuda and T.Kunihiro, Phys.Rep., **247** (1994) 221;

J.A.Miller, B.M.K.Nefkens and I. Slaus, Phys.Rep., **194** (1990) 1-116

[15] C.Quigg, Les Houches Session N.37, 1981, ed. M.K.Gaillard and R.Stora (North-Holland, Amsterdam, 1983)

[16] E.V.Shuryak, Nucl.Phys., **B203** (1982) 93, 116, 140

[17] W.Marciano and H.Pagels, Phys.Rep., **C36** (1978) 139

[18] C.G.Callan, Jr, R.Dashen and D.J.Gross, Phys.Rev., **D17** (1978) 2717

[19] G. 't Hooft, Phys.Rev.Lett., **37** (1976) 8;
D.I.Dyakonov and V.Yu.Petrov, Nucl.Phys., **B272** (1986) 457

[20] R.A.Bertlmann et al, Z.Phys.C-Part.and Fields, **39** (1988) 231;
B.V.Geshkebein and V.LL.Morgunov, Russ.Jour.Nucl.Phys., **58** (1995) 1873

[21] M.-C.Chu, J.M.Grandy, S.Huang and J.W.Negele, Phys.Rev., **D49** (1994) 6039;
T.Schfer, E.V.Shuryak and J.J.M.Verbaarshot, Phys.Rev., **D51** (1995) 1267

[22] V.Gogohia, Gy.Kluge and M.Prisznyák, Preprint hep-ph /9509427

[23] A.Chodos, R.L.Jaffe, K.Johnson, C.B.Torn and V.F.Weisskopf, Phys.Rev., **D9** (1974) 3471

[24] H.H.Gutbrod and J.Rafelski, in: "Particle production in Highly Excited Matter", (Ed. by H.H.Gutbrod and J.Rafelski, NATO ASI Series B: Physics Vol. 303), p.1

TABLES

TABLE I. Calculation scheme A

F_π^0	88.3	92.42	93.3	MeV
Λ_{CSBq}	724.274	758.067	765.284	MeV
m_d	362.137	379.0335	382.642	MeV
$\langle \overline{q}q \rangle_0$	$(-208.56)^3$	$(-218.29)^3$	$(-220.36)^3$	MeV^3
ϵ_q	-0.0012	-0.00143	-0.0015	GeV^4
ϵ_g	-0.0013	-0.00157	-0.0016	GeV^4
ϵ	-0.0025	-0.0030	-0.0031	GeV^4
$\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle$	0.009	0.0106	0.011	GeV^4
m_u^0	6.65	6.36	6.30	MeV
m_d^0	10.08	9.63	9.54	MeV
m_s^0	202.85	193.75	191.94	MeV

TABLE II. Calculation scheme B

$F_\pi^o = 88.3$	$F_\pi^o = 92.42$	$F_\pi^o = 93.3$
$707 \leq \Lambda_c \leq 742.68$	$737.9 \leq \Lambda_c \leq 768.4$	$744.4 \leq \Lambda_c \leq 773.86$
$328.62 \leq m_d \leq 400$	$340 \leq m_d \leq 400$	$342.416 \leq m_d \leq 400$
$(-210.34)^3 \leq \langle \overline{q}q \rangle_0 \leq (-206.9)^3$	$(-219.3)^3 \leq \langle \overline{q}q \rangle_0 \leq (-216.34)^3$	$(-221.2)^3 \leq \langle \overline{q}q \rangle_0 \leq (-218.33)^3$
$-0.00135 \leq \epsilon_q \leq -0.00096$	$-0.0016 \leq \epsilon_q \leq -0.00128$	$-0.0017 \leq \epsilon_q \leq -0.00136$
$-0.0024 \leq \epsilon_g \leq -0.00045$	$-0.00226 \leq \epsilon_g \leq -0.00044$	$-0.00221 \leq \epsilon_g \leq -0.000437$
$-0.00336 \leq \epsilon \leq -0.0018$	$-0.00354 \leq \epsilon \leq -0.002$	$-0.00356 \leq \epsilon \leq -0.0021$
$0.0064 \leq \langle 0|G^2|0 \rangle \leq 0.0128$	$0.007 \leq \langle 0|G^2|0 \rangle \leq 0.0192$	$0.00746 \leq \langle 0|G^2|0 \rangle \leq 0.0199$
$6.48 \leq m_u^0 \leq 6.81$	$6.27 \leq m_u^0 \leq 6.53$	$6.22 \leq m_u^0 \leq 6.47$
$9.83 \leq m_d^0 \leq 10.33$	$9.5 \leq m_d^0 \leq 9.89$	$9.43 \leq m_d^0 \leq 9.81$
$197.67 \leq m_s^0 \leq 207.7$	$191 \leq m_s^0 \leq 199$	$189.76 \leq m_s^0 \leq 197.34$

TABLE III. Calculation scheme A. Predictions

F_π^o	92.42	93.3	MeV
$\epsilon_t = \epsilon_I + \epsilon_g + N_f \epsilon_q$	$-0.00497 - N_f 0.00143$	$-0.005 - N_f 0.0015$	GeV^4
$\langle 0|\frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle$	$0.01767 + N_f 0.00508$	$0.01777 + N_f 0.00533$	GeV^4
B	$0.00497 + N_f 0.00143$	$0.005 + N_f 0.0015$	GeV^4

TABLE IV. Calculation scheme B. Predictions

$F_\pi^o = 92.42$
$-0.00566 - N_f 0.00128 \leq \epsilon_t \leq -0.00384 - N_f 0.0016$
$0.0136 + N_f 0.00568 \leq \langle 0|G^2|0\rangle \leq 0.020 + N_f 0.0045$
$0.00384 + N_f 0.0016 \leq B \leq 0.00566 + N_f 0.00128$

TABLE V. Calculation scheme B. Predictions

$F_\pi^o = 93.3$
$-0.00661 - N_f 0.00136 \leq \epsilon_t \leq -0.003837 - N_f 0.0017$
$0.0136 + N_f 0.006 \leq \langle 0|G^2|0 \rangle \leq 0.0235 + N_f 0.0048$
$0.003837 + N_f 0.0017 \leq B \leq 0.00661 + N_f 0.00136$

FIGURES

FIG. 1. The pion decay constant F_{CA} as a function of k_0, drawn only for the most reasonable region, selected first for the dynamically generated quark masses (3.1). The obtained interval for k_0 is also explicitly shown (see Section 3 below).

FIG. 2. The quark condensate as a function of k_0, drawn only for the most resonable region, selected first for the dynamically generated quark masses (3.1).

FIG. 3. The vacuun energy density due to confining quarks with dynamically generated masses, as a function of k_0, drawn only for the most resonable region, selected first for the dynamically generated quark masses (3.1).

FIG. 4. The vacuum energy density due to the nonperturbative gluons as a function of k_0, drawn only for the most resonable region, selected first for the dynamically generated quark masses (3.1). The obtained interval for k_0 is also explicitly shown (see Section 3 below).

FIG. 5. The vacuun energy density ϵ as a function of k_0, drawn only for the most resonable region, selected first for the dynamically generated quark masses (3.1).

FIG. 6. The vacuum energy density due to the nonperturbative gluons contributions (1.5) as a function of z_0.

FIG. 7. The vacuum energy density due to the nonperturbative gluon contributions ϵ_g as a function of k_0. Λ_c is the confinement scale (3.4). A new interval for m_d (3.3) is also shown. A similar figure can be drawn for the case when the pion decay constant is approximated by the experimental value.

FIG. 8. The pion decay constant F_{CA} as a function of k_0. Λ_c is the confinement scale (3.4). A new interval for m_d (3.3) is also shown. A similar figure can be drawn for the case when the pion decay constant is approximated by the experimental value.

$m_d = 300 MeV$

$m_d = 400 MeV$

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